New BFS expression: (puzzlesPerDist modified to give those with unique paths)

*def* puzzlesPerDist(*puzzle*):  
 p = Puzzle(*puzzle*,0)  
 dictLevel = {0: {*puzzle*}}  
 dictPuzzles = {*puzzle*: [0]}  
 parseMe = [p]  
 *while* parseMe:  
 puzz = parseMe.pop(0)  
 newLevel = puzz.getLevel()+1  
 *for* k *in* neighbors(puzz.getPuzzle()):  
 *if* k *not in* dictPuzzles:  
 newPuzz = Puzzle(k, newLevel)  
 parseMe.append(newPuzz)  
 dictPuzzles[k] = [newLevel]  
 *if* newPuzz.getLevel() *not in* dictLevel:  
 dictLevel[newPuzz.getLevel()] = {k}  
 *else*:  
 dictLevel[newPuzz.getLevel()].add(k)  
 *else*:  
 *try*: #receiving a KeyError:4132\_8765 -- confusing because the key is supposed to be the newLevel  
 *if* newLevel *in* dictPuzzles[k]: #try-except to work around this while I try to understand why  
 dictLevel[newLevel].remove(k)  
 parseMe.append(Puzzle(k, newLevel))  
 *except*:  
 *pass  
 return* dictLevel

Output:

Level: 0 # Puzzles: 1

Level: 1 # Puzzles: 2

Level: 2 # Puzzles: 4

Level: 3 # Puzzles: 8

Level: 4 # Puzzles: 16

Level: 5 # Puzzles: 20

Level: 6 # Puzzles: 38

Level: 7 # Puzzles: 58

Level: 8 # Puzzles: 104

Level: 9 # Puzzles: 140

Level: 10 # Puzzles: 260

Level: 11 # Puzzles: 346

Level: 12 # Puzzles: 656

Level: 13 # Puzzles: 866

Level: 14 # Puzzles: 1578

Level: 15 # Puzzles: 1996

Level: 16 # Puzzles: 3432

Level: 17 # Puzzles: 4056

Level: 18 # Puzzles: 6458

Level: 19 # Puzzles: 6718

Level: 20 # Puzzles: 9756

Level: 21 # Puzzles: 8734

Level: 22 # Puzzles: 10896

Level: 23 # Puzzles: 7400

Level: 24 # Puzzles: 7502

Level: 25 # Puzzles: 3578

Level: 26 # Puzzles: 2562

Level: 27 # Puzzles: 646

Level: 28 # Puzzles: 196

Level: 29 # Puzzles: 6

Level: 30 # Puzzles: 0

Level: 31 # Puzzles: 0

sum: 78033

Other possibly relevant code:

*class* Puzzle():  
 puzzle = ''  
 level = 0  
  
 *def* \_\_init\_\_(self, *puzzle*, *level*):  
 self.puzzle = *puzzle* self.level = *level  
  
 def* getPuzzle(self):  
 *return* self.puzzle  
  
 *def* getLevel(self):  
 *return* self.level

*def* neighbors(*s*):  
 space = *s*.find("\_")  
 lookup = [[1,3],[0,2,4],[1,5],[0,4,6],[1,3,5,7],[2,4,8],[3,7],[4,6,8],[5,7]]  
 neighbors = []  
 *for* i *in* lookup[space]:  
 newS = *s*[0:space] + *s*[i] + *s*[space + 1:]  
 newS = newS[0:i] + "\_" + newS[i+1:]  
 neighbors.append(newS)  
 *return* neighbors

dictLev = puzzlesPerDist("12345678\_")  
sum = 0  
*for* k *in* dictLev:  
 sum = sum + len(dictLev[k])  
 *print*('Level: ', k, ' # Puzzles: ', len(dictLev[k]))  
  
*print*('sum: ', sum)